

MUHANGA DISTRICT

MARKING GUID OF CORE MATHEMATICS S6, TERM II, 2026

1. The area bounded by the curves $y = x^2$ and $y = 2x + 3$ is:

A) 12

B) 18

C) 24

D) 30

Answer is A

2. The volume of the solid formed when the region under $y = \sqrt{x}$ from $x = 0$ to $x = 4$ is revolved about the x-axis is:

A) 8π

B) 16π

C) 32π

D) 64π

Answer is A

3. The work done by a force $F(x) = 3x^2$ as a particle moves from $x = 1$ to $x = 5$ is:

A) 96 J

B) 124 J

C) 150 J

D) 180 J

Answer is B

4. Given the function $f(x) = \frac{e^x - 2}{e^x + 1}$, the value of $f(-3 \ln 2)$

A) $-\frac{3}{5}$

B) $-\frac{5}{3}$

C) $-\frac{5}{8}$

D) $\frac{5}{3}$

Sol

$$sf(x) = \frac{e^{x-2}}{e^{x+1}}$$

$$f(-3 \ln 2) = \frac{e^{-3 \ln 2 - 2}}{e^{-3 \ln 2 + 1}}$$

$$= \frac{e^{\ln 2^{-3} - 2}}{e^{\ln 2^{-3} + 1}}$$

$$= \frac{\frac{1}{8} - 2}{\frac{1}{8} + 1} = -\frac{15}{9} = -\frac{5}{3}$$

Ansuwe is B

5. The length of the arc of the semicubical parabola $y^2 = x^3$ between the point (1, 1) and (4, 8) is

A) $\frac{1}{27}(22\sqrt{22} - 13\sqrt{13})$

B) $\frac{1}{27}(13\sqrt{13} - 22\sqrt{22})$

C) $\frac{1}{27}(8\sqrt{10} - 13\sqrt{13})$

D) $\frac{1}{27}(80\sqrt{10} - 13\sqrt{13})$

sol

For the top half of the curve, we have $y = x^{\frac{3}{2}}$

That means $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$. **1mark**

We calculate the arc of length by using the following technics: $L =$

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad \mathbf{0.5mark}$$

Therefore

$$L = \int_1^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_1^4 \sqrt{1 + \frac{9x}{4}} dx$$

$$u = 1 + \frac{9x}{4} \rightarrow du = \frac{9}{4} dx$$

$$x = 1 \rightarrow u = \frac{13}{4}$$

$$x = 4 \rightarrow u = 10$$

$$\begin{aligned} L &= \frac{4}{9} \int_{\frac{13}{4}}^{10} \sqrt{u} \, du = \frac{4}{9} \times \frac{2}{3} u^{\frac{3}{2}} \Big|_{\frac{13}{4}}^{10} \\ &= \frac{1}{27} (80\sqrt{10} - 13\sqrt{13}) \end{aligned}$$

Answer is D

6. Determine whether the complex number $3+i\sqrt{2}$ satisfies $x^2-6x+11=0$ 4marks

sol

Replace x by $3+i\sqrt{2}$, we obtain: $(3+i\sqrt{2})^2 - 6(3+i\sqrt{2}) + 11$ 1marks $= 9 + 6i\sqrt{2} - 2 - 18 - 6i\sqrt{2} + 11$
1marks $= 0$ 1marks Then, the complex number $3+i\sqrt{2}$ satisfies the equation $x^2-6x+11=0$
1marks

7.a. Determine the order and degree of $x \frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$ 2marks

b. State also if there are linear or non-linear. 1mark

a) $x \frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$

$$x \left(\frac{dy}{dx} \right)^2 + 3 = y^2 \frac{dy}{dx} \quad \text{1mark}$$

The order of the highest derivative is 1 (1mark) and its highest positive integral exponent is 2. 1mark

b) It is non-linear differential equation. 1mark

$\frac{dy}{dx}$ occurs in second degree and also y^2 and $\frac{dy}{dx}$ are multiplied together.

8. Find the average value of the function $f(x) = 1 + x^2$ on the interval $[-1, 2]$

Sol

$$f(x) = 1 + x^2$$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx \quad \text{1mark}$$

$$= \frac{1}{2+1} \int_{-1}^2 (1+x^2) dx \quad \text{1mark}$$

$$= 2 \quad \text{1mark}$$

9. $f(x) = a^x$ passes through point $(\sqrt{3}, 8)$. Find the value of a **2marks**

Sol

$$(\sqrt{3}) = 8 \quad \text{0.5marks}$$

$$a^{\sqrt{3}} = 8 \Leftrightarrow a^{(\sqrt{3})\frac{1}{\sqrt{3}}} = 2^{(3)\frac{1}{\sqrt{3}}} \quad \text{1marks}$$

$$\Leftrightarrow a = 2^{\frac{3}{\sqrt{3}}} = 2^{\sqrt{3}} \quad \text{0.5m}$$

10. Use the Maclaurin's expansion to calculate

$$\text{a) } \lim_{x \rightarrow 0} \frac{1 - \cos 4x + x \sin 3x}{x^2} \quad \text{3marks}$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{\sin^2 x} \quad \text{2marks}$$

sol

$$\text{a) } \lim_{x \rightarrow 0} \frac{1 - \cos 4x + x \sin 3x}{x^2}$$

$$\text{As } x \rightarrow 0, \cos 4x \simeq 1 - \frac{(4x)^2}{2} \simeq 1 - 8x^2 \quad \text{0.5marks}$$

$$\text{and } \sin 3x \simeq 3x \quad \text{0.5marks}$$

Then

$$\lim_{x \rightarrow 0} \frac{1 - \cos 4x + x \sin 3x}{x^2} \quad 0.5 \text{marks}$$

$$= \lim_{x \rightarrow 0} \frac{1 - 1 + 8x^2 + 3x^2}{x^2} \quad 1 \text{marks}$$

$$= 11 \quad 0.5 \text{marks}$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{\sin^2 x}$$

We know that if $x \rightarrow 0$:

$$\ln(1+x) \simeq x - \frac{x^2}{2} \quad 0.5 \text{marks}$$

$$\text{and } \sin^2 x \simeq x^2 \quad 0.5 \text{marks}$$

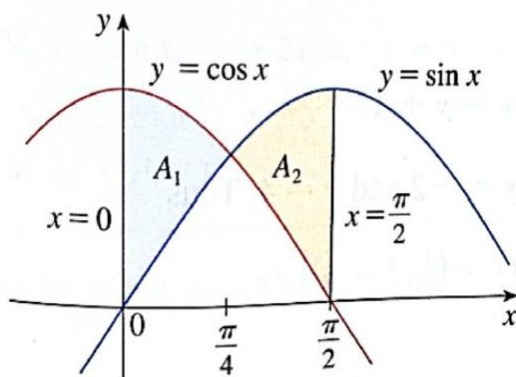
$$\text{Then } \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{x - \frac{x^2}{2} - x}{x^2} \quad 0.5 \text{marks}$$

$$= -\frac{1}{2} \quad 0.5 \text{marks}$$

11. Find the area of the region bounded by the curves $y = \sin x$ and

$$y = \cos x \text{ where } 0 \leq x \leq \frac{\pi}{2} \quad 4 \text{mark}$$

Sol.



$$A = \int_0^{\frac{\pi}{2}} |\cos x - \sin x| dx = A_1 + A_2 \quad 1 \text{mark}$$

$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x) dx \quad \text{1mar}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 \right) + \left(0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \quad \text{1mark}$$

$$= 2\sqrt{2} - 1 \text{ squire unit.} \quad \text{1mark}$$

12. Find the differential equation corresponding to $y^2 = a(b - x^2)$ by eliminating a and b . (4 marks)

Sol

$$y^2 = a(b - x^2)$$

$$\Leftrightarrow y^2 = ab - ax^2 \quad \text{1 mark}$$

Differentiate both sides:

$$2y \frac{dy}{dx} = -2ax \Rightarrow y \frac{dy}{dx} = -ax \quad \text{1 mark}$$

Again, differentiate both sides:

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = -a \quad \text{1 mark}$$

$$\Rightarrow \frac{dy}{dx} = x \left[y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right] \quad \text{1 mark which is the required differential equation.}$$

13. Solve $\int \frac{e^x+1}{e^x-1} dx$ 4marks

Sol;

$$I = \int \frac{e^x + 1}{e^x - 1} dx = \int \frac{(e^x - 1) + 2}{e^x - 1} dx \quad \text{1marks}$$

$$= \int dx + 2 \int \frac{dx}{e^x - 1} \quad \text{1marks}$$

$$= \int dx + 2 \int \frac{e^{-x}}{1 - e^{-x}} dx \quad \text{1marks}$$

$$= x + 2 \ln|1 - e^{-x}| + c \quad \text{1marks}$$

14. Given the function $f: R \rightarrow R: x \rightarrow \log_x \sqrt{1 - x^2}$. Find the domain. **2marks**

Sol

$$f(x) = \log_x \sqrt{1 - x^2}$$

Condition: $1 - x^2 > 0, x \neq 1, x > 0$ **1marks**

x	$-\infty$		-1		1		$+\infty$
$1 - x^2$	$-$	$-$	0	$+$	0	$-$	$-$

Domf: $x \in (0, 1)$ **1marks**

15. Find the value of c if $\lim_{x \rightarrow \infty} \left(\frac{x+c}{x-c} \right)^x = 3$ **4marks**

Solution

$$\lim_{x \rightarrow \infty} \left(\frac{x+c}{x-c} \right)^x = 3$$

$$\Leftrightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{2c}{x-c} \right)^x = 3 \quad \text{1mark}$$

$$x - c = 2ct \Leftrightarrow x = 2ct + c$$

$$x \rightarrow \infty, t \rightarrow \infty$$

$$\lim_{t \rightarrow \infty} \left(1 + \frac{1}{t} \right)^{2ct+c} = 3 \quad \text{1mark}$$

$$\Leftrightarrow \lim_{t \rightarrow \infty} \left[\left(1 + \frac{1}{t} \right)^t \right]^{2c} \left(1 + \frac{1}{t} \right)^c = 3 \quad \text{0.5marks}$$

$$\Leftrightarrow e^{2c} \times 1 = 3 \quad \text{0.5marks}$$

$$\Leftrightarrow 2c = \ln 3$$

$$\rightarrow c = \frac{\ln 3}{2} \quad \text{1mark}$$

SECTION B;

16.

a) Calculate the integral $\int_0^{2\pi} |\sin x| dx$ (5 marks)

b) Using integral rules, Compute the circumference of a circle centred at the origin. (10 marks)

Sol

a) $\int_0^{2\pi} |\sin x| dx$

$$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \geq 0 \\ -\sin x & \text{if } \sin x < 0 \end{cases} \quad \mathbf{1 \text{ mark}}$$

$$\Rightarrow \int_0^{2\pi} |\sin x| dx = \int_0^{\pi} \sin x dx - \int_{\pi}^{2\pi} \sin x dx \quad \mathbf{1 \text{ mark}}$$

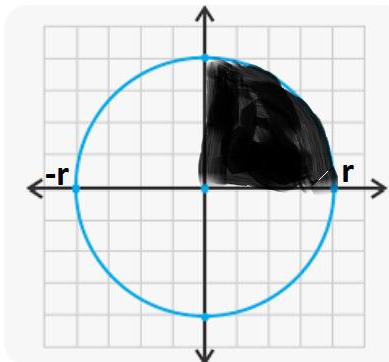
$$\Leftrightarrow \int_0^{2\pi} |\sin x| dx = -\cos x \Big|_0^{\pi} + \cos x \Big|_{\pi}^{2\pi} \quad \mathbf{1 \text{ mark}}$$

$$\Leftrightarrow \int_0^{2\pi} |\sin x| dx = -\cos \pi - (-\cos 0) + [\cos 2\pi - \cos \pi] \quad \mathbf{1 \text{ mark}}$$

$$\Leftrightarrow \int_0^{2\pi} |\sin x| dx = 1 + 1 + 1 + 1 \quad \mathbf{0.5 \text{ marks}}$$

$$\Rightarrow \int_0^{2\pi} |\sin x| dx = 4 \text{ U.A} \quad \mathbf{0.5 \text{ marks}}$$

b)



Consider the arc of the circumference when $y > 0$ and $x > 0$.

$$x^2 + y^2 = r^2 \quad \mathbf{0.5 \text{ marks}}$$

$$y = \pm\sqrt{r^2 - x^2} \text{ 0.5 marks}$$

$$y' = \frac{-x}{\sqrt{r^2 - x^2}} \text{ 0.5 marks}$$

$$\text{Arc} = \int_a^b \sqrt{1 + (y')^2} dx \text{ 0.5 marks}$$

$$\Leftrightarrow \text{Arc} = \int_0^r \sqrt{1 + \left(\frac{-x}{\sqrt{r^2 - x^2}}\right)^2} dx \text{ 0.5 marks}$$

$$\Leftrightarrow \text{Arc} = \int_0^r \sqrt{1 + \frac{x^2}{r^2 - x^2}} dx \text{ 0.5 marks}$$

$$\Leftrightarrow \text{Arc} = \int_0^r \sqrt{\frac{r^2 - x^2 + x^2}{r^2 - x^2}} dx \text{ 0.5 marks}$$

$$\Leftrightarrow \text{Arc} = r \int_0^r \frac{dx}{\sqrt{r^2 - x^2}} \text{ 0.5 marks}$$

$$\Leftrightarrow \text{Arc} = r \arcsin \frac{x}{r} \Big|_0^r \text{ 0.5 marks}$$

$$\Rightarrow \text{Arc} = r \arcsin 1 - 0 = \frac{r\pi}{2} \text{ Units of length 0.5 marks}$$

Hence the total length is

$$C = 4\text{Arc} = 4 \times \frac{r\pi}{2} \text{ Units of length} = 2r\pi \text{ Units of length 1 mark}$$

17 a) Show that $y = Ae^{-2x} + Be^x$ is a general solution of the differential equation : $y'' + y' - 2y = 0$ and find the particular solution that satisfies the condition $y = y' = 1$ when $x = 0$ (8 marks)

b) Calculate the area of the surface bounded by the curve:

$$x = 6(\theta - \sin\theta) \text{ and } y = 6(1 - \cos\theta) \text{ , where } 0 \leq t \leq 2\pi \text{ (7 marks)}$$

$$y = Ae^{-2x} + Be^x$$

$$\Leftrightarrow y' = -2Ae^{-2x} + Be^x \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow y'' = 4Ae^{-2x} + Be^x \dots\dots\dots 1 \text{ marks}$$

$$y'' + y' - 2y = 0 \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow 4Ae^{-2x} + Be^x + (-2Ae^{-2x} + Be^x) - 2(Ae^{-2x} + Be^x) = 0 \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow 4Ae^{-2x} + Be^x - 2Ae^{-2x} + Be^x - 2Ae^{-2x} - 2Be^x = 0 \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow 4Ae^{-2x} - 4Ae^{-2x} + 2Be^x - 2Be^x = 0$$

$$\begin{cases} 1 = A + B \\ 1 = -2A + B \end{cases} \dots\dots\dots 1 \text{ marks}$$

$$A = 0 \text{ and } B = 1 \dots\dots\dots 1 \text{ marks}$$

$$y = e^x \dots\dots\dots 1 \text{ marks}$$

B) $x = 6(\theta - \sin \theta)$ and $y = 6(1 - \cos \theta)$

$$0 \leq \theta \leq 2\pi$$

$$A = \int_a^b f(x) dx = \int_a^b y dx$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta} (6\theta - 6 \sin \theta) = 6(1 - \cos \theta) d\theta \dots\dots\dots 1 \text{ marks}$$

$$A = \int_0^{2\pi} 36(-\cos \theta)(-\cos \theta) d\theta \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow A = \int_0^{2\pi} 36(-\cos \theta)^2 d\theta \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow A = 36 \int_0^{2\pi} (1 - 2 \cos \theta + \cos^2 \theta) d\theta \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow A = 36 \int_0^{2\pi} \left(1 - 2 \cos \theta + \frac{1 + \cos 2\theta}{2} \right) d\theta \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow A = 36 \int_0^{2\pi} \left(\frac{3}{2} - 2 \cos \theta + \frac{\cos 2\theta}{2} \right) d\theta \dots\dots\dots 1 \text{ marks}$$

$$\Leftrightarrow A = 36 \left[\frac{3}{2} \theta - 2 \sin \theta + \frac{1}{4} \sin 2\theta \right]_0^{2\pi} \dots\dots\dots \mathbf{0.5 \text{ marks}}$$

$$\Rightarrow A = 108\pi \text{ units of area} \dots\dots\dots \mathbf{0.5 \text{ marks}}$$

18. Consider the polynomial $P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$, with a_0, a_1, a_2, a_3 coefficients of the polynomial. Given that:

$$P(1+2i) = 0, \quad P(2) = 0, \quad P(0) = 20$$

Determine the coefficients of $P(x)$. **(15 marks)**

Sol $P(0) = 20 \Rightarrow a_0 = 20 \dots\dots\dots \mathbf{2 \text{ marks}}$

$$P(1+2i) = 0 \Rightarrow a_3(1+6i-12-8i) + a_2(1+4i-4) + a_1(1+2i) + 20 = 0$$

$$(-11-2i)a_3 + (-3+4i)a_2 + (1+2i)a_1 + 20 = 0 \dots\dots\dots(2) \quad \mathbf{1 \text{ mark}}$$

$$P(1-2i) = 0 \Rightarrow a_3(1-6i-12+8i) + a_2(1-4i-4) + a_1(1-2i) + 20 = 0$$

$$(-11+2i)a_3 + (-3-4i)a_2 + (1-2i)a_1 + 20 = 0 \dots\dots\dots(3) \quad \mathbf{1 \text{ mark}}$$

(Conjugates zero theorem of polynomials)

$$P(2) = 0 \Rightarrow 8a_3 + 4a_2 + 2a_1 + 20 = 0$$

$$\dots\dots\dots(4) \quad \mathbf{1 \text{ mark}}$$

From (1),(2),(3) and (4) above, we get:

$$\begin{cases} (-11-2i)a_3 + (-3+4i)a_2 + (1+2i)a_1 = -20 \\ (-11+2i)a_3 + (-3-4i)a_2 + (1-2i)a_1 = -20 \quad (\xi) \mathbf{2 \text{ marks}} \\ 8a_3 + 4a_2 + 2a_1 = -20 \end{cases}$$

Solving the system of linear equation in (ξ) (Crammer's method "Sarrus expansion"), we get:

$$\Delta = -200i \quad \Delta_1 = 3600i \quad \Delta_2 = -1600i \quad \Delta_3 = 400i \quad \mathbf{4 \text{ marks}}$$

$$\therefore a_0 = 20, \quad a_1 = -18, \quad a_2 = 8, \quad a_3 = -2 \quad \mathbf{3 \text{ marks}}$$

$$P(x) = -2x^3 + 8x^2 - 18x + 20 \quad \mathbf{1 \text{ mark}}$$

19. $f(x) = xe^{-x}$

- Find the domain of $f(x)$ **(1 mark)**
- Determine the parity of $f(x)$ **(2 marks)**
- Find the intersection points of $f(x)$ **(1 mark)**
- Determine the limit on bounding points and asymptotes of $f(x)$ **(4 marks)**
- Find the variation tables of $f(x)$ **(6 marks)**
- Sketch the graph of $f(x)$ **(1 mark)**

$$f(x) = xe^{-x}$$

- Dom $f(x) = \mathbb{R}$ **1 mark**
- Parity: $f(-x) = -xe^x \neq f(x)$ **0.5 marks**
 $-f(-x) = xe^x \neq f(x)$ **0.5 marks**
 Neither even nor odd **1 mark**

- Intersecting points:
 $C \cap 0X \equiv \{(0,0)\}$ **0.5 marks**
 $C \cap 0Y \equiv \{(0,0)\}$ **0.5 marks**

- Limits on bounding points and asymptotes:

$$VA \equiv x = a \Big|_{a \text{ point on boundary}} : \lim_{x \rightarrow a} f(x) = \infty : \text{not admitted.} \mathbf{1 \text{ mark}}$$

$$HA \equiv y = b \Big|_{b \in \mathbb{R}} : \lim_{x \rightarrow \infty} f(x) = b \Rightarrow \lim_{x \rightarrow \infty} xe^{-x} = \lim_{x \rightarrow \infty} \frac{x}{e^x} = \frac{\infty}{\infty} : \mathbf{1 \text{ mark}} \text{ Indeterminate case}$$

By Hospital rule,

$$\lim_{x \rightarrow \infty} \frac{x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0 \mathbf{0.5 \text{ marks}}$$

$$HA \equiv y = 0 \mathbf{0.5 \text{ marks}}$$

No slant asymptote admitted. **1 mark**

e) Variation table :

Step 1: Study of f' :

i) $f'(x) = (xe^{-x})' = e^{-x} - xe^{-x} = e^{-x}(1-x)$ **0.5 marks**

ii) $f'(x) = 0 \Rightarrow x = 1$:critical point of the function(at $x = 1$,the tangent line is horizontal)**0.5 marks**

iii)T.V of f for f' and extrema points**0.5 marks**

x	$-\infty$	1	$+\infty$
$f'(x)$	+++++0-----		
$f(x)$			

1 mark

Step 2: Study of f'' :

i) $f''(x) = [(1-x)e^{-x}]' = (x-2)e^{-x}$ **0.5 marks**

ii) $f''(x) = 0 \Rightarrow x = 2$, inflection point is $(2, \frac{2}{e^2})$ **0.5 marks**

iii)T.V of f for f'' and nature of concavity:**0.5 marks**

x	$-\infty$	2	$+\infty$
$f''(x)$	-----0+++++		
$f(x)$			

1 mark

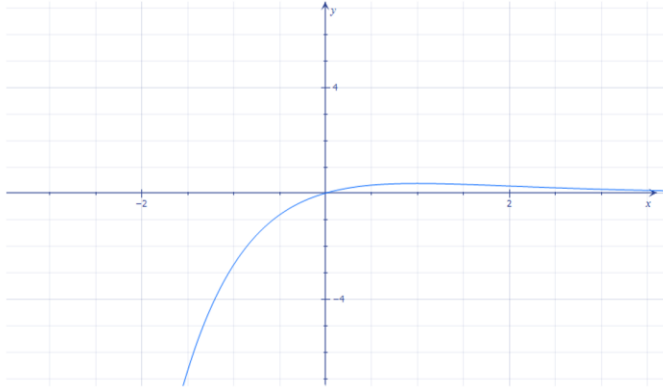
Step 3: Table of variation of f

x	$-\infty$	1	2	$+\infty$
$f'(x)$	+++++0-----			
$f''(x)$	-----0+++++			
$f(x)$				

1 mark

e) Graph

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	$-4e^4$	$-3e^3$	$-2e^2$	$-e$	0	e^{-1}	$2e^{-2}$	$3e^{-3}$	$4e^{-4}$



1 mark

20. Find the particular solution of the differential equation:

$$7x(x - y)dy = 2(x^2 + 6xy - 5y^2)dx \text{ given that } x = 1 \text{ when } y = 0$$

15marks

$$\text{Sol } 7x(x - y)dy = 2(x^2 + 6xy - 5y^2)dx$$

the differential equation is homogeneous

$$\frac{dy}{dx} = \frac{2(x^2 + 6xy - 5y^2)}{7x(x - y)} \quad \text{1mark}$$

$$\text{let } y = ux \rightarrow \frac{dy}{dx} = x \frac{du}{dx} + u \quad \text{1mark}$$

$$x \frac{du}{dx} + u = \frac{2(x^2 + 6x(ux) - 5(ux)^2)}{7x(x - ux)} \quad \text{1mark}$$

$$= \frac{2x^2(1 + 6u - 5u^2)}{7x^2(1 - u)} = \frac{2(1 + 6u - 5u^2)}{7(1 - u)} \quad \text{1mark}$$

$$x \frac{du}{dx} = \frac{2(1 + 6u - 5u^2)}{7(1 - u)} - u \quad \text{0.5mark} = \frac{2(1 + 6u - 5u^2) - 7(1 - u)u}{7(1 - u)} \quad \text{0.5mark}$$

$$= \frac{2 + 12u - 10u^2 - 7u + 7u^2}{7(1 - u)} = \frac{-3u^2 + 5u + 2}{7(1 - u)} \quad \text{0.5mark}$$

$$\frac{7(1-u)}{-3u^2+5u+2} du = \frac{dx}{x} \quad \text{0.5mark}$$

$$\int \frac{7(1-u)}{-3u^2+5u+2} du = \int \frac{dx}{x} \quad \text{0.5mark}$$

$$\frac{7(1-u)}{-3u^2+5u+2} = \frac{7(1-u)}{-(u-2)(3u+1)} = \frac{-7(1-u)}{(u-2)(3u+1)} \quad \text{0.5mark}$$

$$= \frac{a}{u-2} + \frac{b}{3u+1} = \frac{a(3u+1)+b(u-2)}{(u-2)(3u+1)} \quad \text{0.5mark}$$

$$-7(1-u) = a(3u+1) + b(u-2) \quad \text{0.5mark}$$

Let $u = 2$, then $-7(1-2) = a(6+1)$ **0.5mark**

,

$$7a = 7 \rightarrow a = 1 \quad \text{0.5mark}$$

Let $u = -\frac{1}{3}$, then $-7\left(1 + \frac{1}{3}\right) = b\left(-\frac{1}{3} - 2\right)$, **0.5mark**

$$-\frac{7}{3}b = \frac{-28}{3}, \rightarrow b = 4 \quad \text{0.5mark}$$

$$\frac{7(1-u)}{-3u^2+5u+2} = \frac{1}{u-2} + \frac{4}{3u+1} \quad \text{0.5mark}$$

$$\int \frac{7(1-u)}{-3u^2+5u+2} du = \int \frac{dx}{x} \quad \text{0.5mark}$$

$$\int \frac{7(1-u)}{-3u^2+5u+2} du = \int \frac{1}{u-2} + \frac{4}{3u+1} du \quad \text{0.5mark}$$

$$= \ln(u-2) + \frac{4}{3} \ln(3u+1) \quad \text{1mark}$$

$$\ln\left((u-2)(3u+1)^{\frac{4}{3}}\right) = \ln(Ax) \quad \text{0.5mark}$$

$$(u-2)(3u+1)^{\frac{4}{3}} = Ax \quad \text{0.5mark}$$

or $y = ux \rightarrow u = \frac{y}{x}$

$$\left(\frac{y}{x} - 2\right) \left(3\frac{y}{x} + 1\right)^{\frac{4}{3}} = Ax \quad \text{0.5mark}$$

$$\leftrightarrow \left(\frac{y-2x}{x}\right) \left(\frac{3y+x}{x}\right)^{\frac{4}{3}} = Ax \quad \text{0.5mark}$$

When $x = 1$ $y = 0$, $\left(\frac{0-2}{1}\right) \left(\frac{0+1}{1}\right)^{\frac{4}{3}} = A \rightarrow A = -2$, **1mark**

Thus, the particular solution is

$$\left(\frac{y-2x}{x}\right) \left(\frac{3y+x}{x}\right)^{\frac{4}{3}} = -2x \quad \text{0.5mark}$$