

MUHANGA DISTRICT

ADVANCED LEVEL EXAMINATIONS, 2026

SUBJECT: MATHEMATICS S5

COMBINATIONS:

- MATH-CHEMISTRY-BIOLOGY(MCB)
- MATH-ECONOMICS-GEOGRAPHY(MEG)
- MATH-PHYSICS-GEOGRAPHY(MPG)
- PHYSICS-CHEMISTRY-MATH(PCM)
- MATH-COMPUTER- ECONOMY(MCE)
- MATH- PHYSICS-COMPUTER(MPC)

DURATION: 3 HOURS

INSTRUCTIONS:

This paper has **2** sections **A** and **B**

SECTION A: Attempt **ALL** questions **(55 marks)**

SECTION B : Attempt any **THREE** questions **(45 marks)**

SECTION A: Attempt all the questions **(55 marks)**

1. Answer by True(T) or False(F)/**4 Marks**

- a) A square matrix A and its transpose A^t have the same determinant
- b) The natural logarithm of negative number does not exist
- c) An invertible matrix has determinant of zero
- d) $\sin 2A = 2 \sin A \cos A$

2. Choose the letter corresponding to the right alternative:

a) The 2nd term of a harmonic sequence is $\frac{1}{9}$ and the 4th term is $\frac{1}{17}$. The 1st term is:

- i) $\frac{1}{7}$
- ii) 5
- iii) $\frac{1}{5}$
- iv) $\frac{1}{3}$

/2marks

b) An arithmetic progression of eleven terms whose first term is 1 and whose last is 6 has

a sum of: i) $\frac{77}{2}$ ii) 7 iii) $\frac{7}{2}$ iv) 12 **/2 marks**

3. Find the fundamental period of the function $f(t) = \tan\left(\frac{1+t}{2}\right) \sin\left(\frac{2t+1}{5}\right)$ **/3marks**

4. Simplify the following trigonometric inverses:

a) $\cos(\sin^{-1}x)$ **/2marks**

b) $\sec^2(\tan^{-1}x)$ /2marks

5. The value of x in $\begin{vmatrix} 11-x & 2 & 8 \\ 2 & 2-x & -10 \\ 8 & -10 & 5-x \end{vmatrix} = 0$ is:

A(9,3,-9) B(-9,9,18) C(-9,9,17) D. None of the above /5marks

6. Light travels from air into an optical fiber with an index of refraction of 1.44

a) In which direction does the light bend? /2marks

b) If the angle of incidence on the end of the fiber is 22° , what is the angle of refraction inside the fiber /3marks

7. Anthony invest Rwf at a rate of interest 4% per annum. After 5 years, it will be worth Rwf 20000. How much will it be worth after 10 years? /4marks

8. Given that $2 \sin\theta = 7 \cos\theta$. Find the value of $\cot\theta$ /2MARKS

9. Find n^{th} derivative of $y = \cos x$ /3marks

10. Determine the direction cosines of vector with component (1,2, -3) /3marks

11. The definition of the gradient of a line says that: The gradient of a line is a measure of its...../2marks

- a) stepness
- b) steepness
- c) steepness
- d) stipness

12. Find the derivative of $f(x) = \sin^{-1}x^3$ /3marks

13. let $A = 4\sin^2x + 2\cos^2x - 3$ with $x \in (-\pi, \pi)$

a) Express A in terms of $\cos 2x$ /3marks

b) Determine the value of x for $A = \frac{1}{2}$ /2marks

14. For what value of m is the vector $\vec{v}(1, -2, m)$ of \mathbb{R}^3 a linear combination of vectors $\vec{u}(3,0,-2)$ and $\vec{w}(2,-1,-5)$

/4marks

15. Find the volume of tetrahedron whose vertices are the points A(3,2,1), B(1,2,4), C(4,0,3) and D(1,1,7) /4marks

SECTION B ATTEMPT ANY THREE QUESTIONS(45 MARKS)

16. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ -1 & 0 & -2 \end{bmatrix}$

a) Show that $A^3 - 4A + 7I = 0$ where I, 0 are the unit and null matrices of order 3 respectively. Use this result to find A^{-1} (10 marks)

b) Find the matrix x such that $AX = \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix}$ (5 marks)

17. Find the matrix representation of the transformation $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$f(x, y, z) = (3x + 2y, 2z - y, z - x)$$

a) Relative to the standard basis of \mathbb{R}^3 (5marks)

b) Relative to the basis $\{\vec{e}_1 = (1, 1, 1), \vec{e}_2 = (-1, 0, 1), \vec{e}_3 = (0, 1, 1)\}$ (10marks)

18. The equation $S = 2 - 2\sin^2 3t$ gives the position of a body moving on a coordinate line (s in meters and t in seconds)

a. Find the body's speed at time $t = \frac{\pi}{4}$ sec (5marks)

b. Find the body's acceleration at time $t = \frac{\pi}{4}$ sec (5marks)

c. Find the body's jerk at time $t = \frac{\pi}{4}$ sec (5marks)

19. The sum of three numbers forming a geometric progression is 21 and the sum of their squares is 189. Find the three numbers (15Marks)

20. a) Given that $f(x) = -1 + \tan^{-1}\left(\frac{4x}{5}\right)$, find the inverse function $f^{-1}(x)$. (5marks)

b) calculate $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ (10 marks)

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