Correction of examination S1 2026

Marking guide of S1 Math Muhanga District

Section A

(1)

a) T /1 marks
b) T /1 marks
c) F /1 marks
d) T /1 mark

(2) a) 2nd term $= \frac{1}{9}$ harmonic sequence is the
4th term $= \frac{1}{17}$ reciprocal of A.p

let the corresponding A.p be $a, a+d, a+2d, a+3d$

$a+d=9, \quad a+3d=17$

$a+3d-(a+d)=17-9$

$2d=8 \Rightarrow d=4$

$a+d=9$

$a=9-4$

$a=5$

harmonic sequence is $\frac{1}{5}$

Correct answer is iii) $= \frac{1}{5}$ /2 marks

(3) number of terms $n=11$, 1st term $=1$, last term $=6$

$S_n = \frac{n}{2}(U_1+U_n)$

$S_{11} = \frac{11}{2}(1+6)$ the correct answer is i) $\frac{77}{2}$

$S_n = \frac{77}{2}$ /2 marks

③ $f(t) = \tan\left(\frac{t+1}{2}\right) \sin\left(\frac{2t+1}{5}\right)$

for $f(t) = \tan\left(\frac{t+1}{2}\right)$

$\tan\left(\frac{t+1}{2}\right) + \pi = \tan \frac{1}{2}(t+p) + \frac{1}{2}$

$p = 2\pi$ / 1 marks

for $f(t) = \sin\left(\frac{2t+1}{5}\right)$

$\sin\left(\frac{2t+1}{5}\right) + 2\pi = \sin\left(\frac{2}{5}(t+p) + \frac{1}{5}\right)$

$p = 5\pi$ / 1 marks

$P(2\pi, 5\pi) = P(10\pi) \Rightarrow P = 10\pi$ / 1 mark.

④ a) $\cos(\sin^{-1}x)$

let $\theta = \sin^{-1}x \Rightarrow \sin\theta = x$

$\cos^2\theta + \sin^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$\cos\theta = \sqrt{1 - \sin^2(\sin^{-1}x)} \Rightarrow \cos(\sin^{-1}x) = \sqrt{1-x^2}$ / 2 marks

b) $\sec^2(\tan^{-1}x) \Rightarrow$ let $\theta = \tan^{-1}x \Rightarrow \tan\theta = x$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2(\tan^{-1}x) = 1 + x^2$ / 2 marks

⑤ $\begin{vmatrix} 11-x & 2 & 8 \\ 2 & 2-x & -10 \\ 8 & -10 & 5-x \end{vmatrix} = 0$

$$(11-x)\begin{vmatrix} 2-x & -10 \\ -10 & 5-x \end{vmatrix} - 2\begin{vmatrix} 2 & -10 \\ 8 & 5-x \end{vmatrix} + 8\begin{vmatrix} 2 & 2-x \\ 8 & -10 \end{vmatrix} = 0$$

$x^3 - 18x^2 - 81x + 1458 = 0$

$x = 9$

$(x-9)(x^2 - 9x - 162) = 0$

$x = 18$ or $x = -9$

$x = 9$

The Correct answer is $B(-9, 9, 18)$ / 5marks

⑥ a) Since the light is traveling from a rarer region to a denser region it will bend toward the normal / 2marks

b) $n_1 = 1$, $n_2 = 1.44$ and $\theta_1 = 22°$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_2 = \frac{\sin 22}{1.44}$

$\theta_2 = \sin^{-1}(0.26) \Rightarrow \theta_2 = 15°$ / 3marks

⑦ $A = 20000$ $t = 5$ years $r = 0.04$

$20000 = P(1+0.04)^5$

$P = \frac{20000}{1.21665} = 16439.18\,F$

At $t = 10$ years

$A = 16439.18 \times (1.04)^{10}$

$A = 24333.06\,F$ / 4marks)

⑧ $y = \cos x$

$y' = -\sin x = \cos(x + \frac{\pi}{2})$ $y^{(n)} = \cos(x + \frac{n\pi}{2})$ / 3 marks

$y'' = -\cos x = \cos(x + \frac{2\pi}{2})$

$y''' = \sin(x + \frac{3\pi}{2})$

(9) $2\sin\theta = 7\cos\theta$

$2 = \frac{7\cos\theta}{\sin\theta}$

$\frac{2}{7} = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \frac{2}{7}$ / 3 marks

(10) $\cos\alpha = \frac{1}{\sqrt{1^2+2^2+(-3)^2}} = \frac{1}{\sqrt{14}}$ / 1 marks

$\cos\beta = \frac{2}{\sqrt{14}}$ / 1 marks

$\cos\gamma = \frac{-3}{\sqrt{14}}$ / 1 marks

(11) b) steepness / 2 marks

(12) $\sin^{-1} x^3$

$f'(x) = \frac{(x^3)'}{\sqrt{1-(x^3)^2}} = \frac{3x^2}{\sqrt{1-x^6}}$ / 3 marks

(13) a) $A = 4\sin^2 x + 2\cos^2 x - 3$

$A = 4\left(\frac{1-\cos 2x}{2}\right) + 2\left(\frac{1+\cos 2x}{2}\right) - 3$

$A = 2 - 2\cos 2x + 1 + \cos 2x - 3$

$A = -\cos 2x$ / 3 marks

b) $-\cos 2x = 0 \Leftrightarrow 2x = \pm\frac{\pi}{2} + 2k\pi$

$x = \pm\frac{\pi}{4} + k\pi$ / 2 marks

(14) $V = c_1\vec{u} + c_2\vec{w}$

$$V = c_1\begin{pmatrix}3\\0\\-2\end{pmatrix} + c_2\begin{pmatrix}2\\-1\\-5\end{pmatrix}$$

$3c_1 + 2c_2 = 1$

$-c_2 = -2$

$c_2 = 2$

$c_1 = -1$

$-2c_1 - 5c_2 = m$

$-2(-1) - 5(2) = m$

$m = -8$ / 4 marks

(15) $\vec{AB} = (-2, 0, 3)$, $\vec{AC} = (1, -2, 2)$, $\vec{AD} = (-2, -1, 6)$

the $V = \frac{1}{6}\begin{vmatrix}-2 & 0 & 3\\ 1 & -2 & 2\\ -2 & -1 & 6\end{vmatrix}$

$V = \frac{5}{6}$ cubic units / 4 marks

SECTION B

(16) $A = \begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 2\\ -1 & 0 & -2\end{bmatrix}$

$$A^2 = \begin{pmatrix}1 & 2 & -1\\ 0 & 1 & 2\\ -1 & 0 & -2\end{pmatrix}\begin{pmatrix}1 & 2 & -1\\ 0 & 1 & 2\\ -1 & 0 & -2\end{pmatrix} = \begin{pmatrix}2 & 4 & 5\\ -2 & 1 & -2\\ 1 & -2 & 5\end{pmatrix}$$

$$A^3 = A \cdot A^2 = \begin{pmatrix}1 & 2 & -1\\ 0 & 1 & 2\\ -1 & 0 & -2\end{pmatrix}\begin{pmatrix}2 & 4 & 5\\ -2 & 1 & -2\\ 1 & -2 & 5\end{pmatrix} = \begin{pmatrix}-3 & 8 & -4\\ 0 & -3 & 8\\ -4 & 0 & -15\end{pmatrix}$$

$$4A = \begin{bmatrix} 4 & 8 & -4 \\ 0 & 4 & 8 \\ -4 & 0 & -8 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$A^3 - 4A + 7I = \begin{pmatrix} -3 & 8 & -4 \\ 0 & -3 & 8 \\ -4 & 0 & -15 \end{pmatrix} - \begin{pmatrix} 4 & 8 & -4 \\ 0 & 4 & 8 \\ -4 & 0 & -8 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} -3-4+7 & 8-8+0 & -4+4+0 \\ 0-0+0 & -3-4+7 & 8-8+0 \\ -4+4+0 & 0-0+0 & -15+8+7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ / 5 marks

A^{-1}

$$\det A = \begin{vmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ -1 & 0 & -2 \end{vmatrix} = -7$$

find cofactor matrix

- $C_{11} = (+)\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = -2$
- $C_{12} = -\begin{vmatrix} 0 & 2 \\ -1 & -2 \end{vmatrix} -2$
- $C_{13} = +\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 1$

$C_{21} = (-)\begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix} = 4$

$C_{22} = +\begin{vmatrix} 1 & -1 \\ -1 & -2 \end{vmatrix} = -3$

$C_{23} = -\begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} = -2$

$$C_{31} = (+) \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5$$

$$C_{32} = - \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = -2$$

$$C_{33} = + \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$$

cofactor matrix : $CM = \begin{pmatrix} -2 & -2 & 1 \\ 4 & -3 & -2 \\ 5 & -2 & 1 \end{pmatrix}$

$$\operatorname{adj}(A) = C^T = \begin{pmatrix} -2 & 4 & 5 \\ -2 & -3 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

$$A^{-1} = \begin{pmatrix} \frac{2}{7} & \frac{-4}{7} & \frac{-5}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{-1}{7} & \frac{2}{7} & \frac{-1}{7} \end{pmatrix}$$ /5 marks

b) $AX = \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} \Rightarrow X = A^{-1} \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix}$

$$X = \begin{pmatrix} \frac{2}{7} & \frac{-4}{7} & \frac{-5}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{-1}{7} & \frac{2}{7} & \frac{-1}{7} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$$ /5 marks

17. $f(x,y,z) = (3x+2y, 2z-y, z-x)$

for a) $\vec{e}_1(1,0,0)$, $\vec{e}_2(0,1,0)$, $\vec{e}_3(0,0,1)$

$f(\vec{e}_1) = (3, 0, -1)$

$f(\vec{e}_2) = (2, -1, 0)$

$f(\vec{e}_3) = (0, 2, 1)$

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & -1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

/5 marks

b) $f(1,1,1) = (5,1,0)$

$f(-1,0,1) = (-3,2,2)$

$f(0,1,1) = (2,1,1)$

$$\begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + c\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} a-b=5 \\ a+c=1 \\ a+b+c=0 \end{cases}$$

$a=4$ $b=-1$ $c=-3$

$$\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix} = d\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + e\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + f\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} d-e=-3 \\ d+f=2 \\ d+e+f=2 \end{cases}$$

$d=-3$
$e=0$
$f=5$

$$\Rightarrow \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = g\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + h\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + i\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} g - h = 2 \\ g + i = 1 \\ g + h + i = 1 \end{cases} \qquad \begin{aligned} g &= 2 \\ h &= 0 \\ i &= -1 \end{aligned}$$

$$[f]_e = \begin{bmatrix} 4 & -3 & 2 \\ -1 & 0 & 0 \\ -3 & 5 & -1 \end{bmatrix}$$

/10 marks

(18) $s = 2 - 2\sin^2 3t$

a) $v(t) = \dfrac{ds}{dt} = -4 \times 3 \sin 3t \cos 3t$

$v(t) = -12 \sin 3t \cos 3t$

$v(\pi/4) = -12 \sin \dfrac{3\pi}{4} \cos \dfrac{3\pi}{4}$

$v(\pi/4) = 6 \, m/s$

/5 marks

b) $a(t) = \dfrac{d^2s}{d^2t} = (-12 \sin 3t \cos 3t)' = \dfrac{d}{dt}(-6 \sin 6$

$a(t) = -36 \cos 6t)$

$a(\pi/4) = -36 \cos 6(\pi/4)$

$= 0 \, m/s^2$

c) The jerk at $t = \pi/4$

$$J(t) = \frac{d^3s}{d^3t} = (-36\cos 6t)'$$

$$= 216 \sin 6t$$

$$J(\pi/4) = -216\ m/s^3$$

(19) $\frac{x}{r}, x, xr$

$$\frac{x}{r} + x + xr = 21$$

$$\left(\frac{x}{r}\right)^2 + x^2 + (xr)^2 = 189$$

$$x = \frac{21r}{1+r+r^2}$$

$$x^2 = \frac{189r^2}{r^4+r^2+1}$$

$$\left(\frac{21r}{1+r+r^2}\right)^2 = \frac{189r^2}{r^4+r^2+1}$$

$$\frac{441r^2}{r^4+2r^3+3r^2+2r+1} = \frac{189r^2}{r^4+r^2+1}$$

$$f(r) = 252r^6 - 378r^5 - 126r^4 - 378r^3 + 252r^2 = 0$$

$f(2) = 0$ $\quad r = 2$ $\quad u_1 = \frac{x}{r} = \frac{6}{2} = 3$

$x = 6$ $\quad u_2 = x = 6$

$u_3 = 6 \times 2 = 12$

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$S = \{3, 6, 12\}$

(20) a) Let $y = f(x)$

$$y = -1 + \tan^{-1}\left(\frac{4x}{5}\right)$$

$$\tan^{-1}\left(\frac{4x}{5}\right) = y+1$$

$$\tan\left(\tan^{-1}\left(\frac{4x}{5}\right)\right) = \tan(y+1)$$

$$\frac{4x}{5} = \tan(y+1)$$

$$x = \frac{5}{4}\tan(y+1)$$

Then $f^{-1}(x) = \frac{5}{4}\tan(x+1)$ / 5 marks

b) $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3} = \frac{0}{0}$ (I.f)

$$\lim_{x\to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x\to 0} \frac{\sin x(\sec x - 1)}{x^3}$$

$$\lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \left(\frac{\sec x - 1}{x^2}\right)$$

$$= \lim_{x\to 0} \left(\frac{\sec x - 1}{x^2}\right) = \frac{0}{0} \text{ (I.f)}.$$

$$\lim_{x\to 0} \frac{(\sec x - 1)'}{(x^2)'} \Rightarrow \lim_{x\to 0} \frac{\sec x \tan x}{2x}$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin x}{x} \frac{(\sec x)^2}{2} \Rightarrow \lim_{x\to 0} \frac{(\sec x)^2}{2}$$

$$= \frac{1}{2}$$ / 10 marks

End