



HOLYDAYS WORK QUESTION PAPER GRADE:

SENIOR FIVE

COMBINATIONS:

- Mathematics-Computer Science-Economics **(MCE)**
- Mathematics-Economics-Geography **(MEG)**

MARKS:/100

INSTRUCTIONS

- 1) This paper contains **two** sections:
Section A: Attempt **all** questions **(55 marks)**
Section B: Attempt **three** questions only **(45 marks)**
- 2) You may use mathematical instruments and a calculator **where necessary**

Section A: Attempt **all** questions **(55 marks)**

1. Simplify $\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$ **(4 marks)**

2. If $\sin A = -\frac{5}{13}$, $\pi < A < \frac{3\pi}{2}$ and $\cos B = -\frac{3}{5}$, $\frac{\pi}{2} < B < \pi$, Find the value of $\tan(A - B)$ without use of calculator. **(4 marks)**

3. Solve
- a) $|3x - 2| < 1$ (2 marks)
- b) $8^{1-y} = 4^{2y+3}$ (2 marks)
4. Without using Hospital's rule, show that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ (4 marks)
5. Find n^{th} derivative of $y = \cos x$
6. Determine the direction cosines of vector with component $(1, 2, -3)$ (4 marks)
7. Given that $2 \sin \theta = 7 \cos \theta$. Find the value of $\cot \theta$ (4 marks)
8. In the study of a function, a curve is said to be concave upwards (or convex downwards) in the interval $]a, b[$ if
- a) $f''(x) > 0$ for all $x \in]a, b[$
- b) $f''(x) < 0$ for all $x \in]a, b[$ (4 marks)
9. The definition of the gradient of a line says that: "The gradient of a line is a measure of its....." (3 marks)
- a) Stepness
- b) Steepness
- c) Steapness
- d) Stipness
10. Three learners of senior five were in group and discussed the equation of the tangent and the normal of the curve $y = 5(x^2 - 3)^{-1}$ at the point $(2, 5)$. Below are the answers provided by them (3 marks)
- a) Mugisha got $y - 20x - 45 = 0$ and $20y = -x + 98$
- b) Niyomugabo got $y = 20x + 45$ and $-x + 20y = 98$
- c) Irasubiza got $y = 20x - 45$ and $y = \frac{x}{20} + \frac{98}{20}$
11. The first four n^{th} terms of the sequence $\{U_n\}$ where $\frac{1}{\sqrt{5}}, \frac{1+\sqrt{5}}{2}, \frac{1}{\sqrt{5}}, \frac{1-\sqrt{5}}{2}$ (3 marks)
- $U_n = \binom{n}{2} - \sqrt{5} \binom{n}{2}, n \geq 2$ are:
- a) 1,2,3,4
- b) 1,3,5,7
- c) 1,2,3,5
- d) 2,4,6,8
12. Matrices representation of linear transformation f and g are
- $A = \begin{pmatrix} 3 & 4 & 1 \\ -1 & 2 & 0 \\ 4 & -5 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 0 & 3 \\ 1 & 0 & 2 \end{pmatrix}$ respectively. Find the matrix representation of:
- a) $f \circ g$ (2 marks)
- b) $g \circ f$ (2 marks)
13. Find the derivative of $f(x) = \sin^{-1} x^3$ (3 marks)

14. Find the volume of triangular prism whose vertices are the points $A(1,2,1), B(2,4,0), C(-1,2,1)$ and $D(2, -2,2)$. **(3 marks)**

15. The population of a country grow according to the law $P = Ae^{0.06t}$, where P is million in the population at time t and A is a constant given that $t = 0$, the population is 27.3 millions
Calculate the population when:

i. $t = 10$ **(2 marks)**

ii. $t = 15$ **(2 marks)**

Section B: Attempt three questions only (45 marks)

16. The first term of an arithmetic series is 1. The common difference of the series is 6.

a) Find the 10th term of the series **(5 marks)**

b) The sum of the first n terms of the series is 7400.

i. Show that $3n^2 - 2n - 7400 = 0$ **(5 marks)**

ii. Find the value of n . **(5 marks)**

17. If $A = \begin{pmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Verify that $A \cdot \text{Adj}(A) = \det(A) \cdot I$ **(15 marks)**

18. Let T be a linear operator on \mathbb{R}^3 defined by

$$T(x, y, z) = (2y + z, x - 4y, 3x)$$

a) Find the matrix of T in the basis $\{e^{\rightarrow}_1 = (1,1,1), e^{\rightarrow}_2 = (1,1,0), e^{\rightarrow}_3 = (1,0,0)\}$ **(8 marks)**

b) Verify that $[T]_{e.} [v^{\rightarrow}]_e = [T(v^{\rightarrow})]_e$ for any vector $v^{\rightarrow} \in \mathbb{R}^3$. **(7 marks)**

19. (a) Evaluate $\lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2 \cos x}{x - \frac{\pi}{3}}$ **(5 marks)**

(b) Simplify the function $\sec^2(\tan^{-1}x)$ **(5 marks)**

(c) For which values of x is true that $\csc(\csc^{-1}x) = x$ **(5 marks)**

20. (a) Solve graphically $x - 2 \sin x = 0$ **(6 marks)**

(b) Find the fundamental period of $f(x) = \cos x\sqrt{3} + \sin 6x$ **(9 marks)**

Good Luck!!!!